EIT/EIA Resonances Driven by the Light Field of Elliptically Polarized Waves

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***Abstract*—Outside of the perturbation theory for bichromatic field of two co-propagating elliptically polarized waves, the type of the subnatural (electromagnetically induced transparency (EIT) or absorption (EIA)) resonance is theoretically found to determined only by the angular momenta of the ground (F_g) and the excited (F_e) states of atomic gas. Herewith, the spontaneous transfer of anisotropy from the excited state to the ground one leads to the formation of the EIA resonance at the transition $F_g = F \to F_e = F + 1$.**

Keywords—electromagnetically induced transparency, electromagnetically induced absorption, anisotropy transfer, low-frequency Zeeman coherence

I. Introduction

Nonlinear interference effects based on atomic coherence are of great interest in modern laser spectroscopy. An example of such effects are the resonances of electromagnetically induced transparency (EIT) [1] and absorption (EIA) [2]. The first type of resonance is associated with the phenomenon of coherent population trapping (CPT) [3], when under certain conditions the electromagnetic field ceases to interact with the atomic medium and a long-lived coherent state (dark state) and ultra-narrow absorption dip are formed. In turn, the physical cause of the EIA resonance, which is opposite in sign to the EIT resonance, is the spontaneous transfer of the anisotropy (including the low-frequency Zeeman coherence) from the excited state of the atom to the ground one [4]. The main feature of such resonances is their width, which can be much less than natural and reaches hundreds and units of Hz [5, 6]. Therefore, they find many interesting applications in the field of quantum metrology [7, 8], nonlinear optics, optical communications, etc.

For these resonances, the question of their sign (type) is important. Currently, due to various experimental [2, 9, 10] and theoretical investigations [11, 12], the following classification of atomic dipole transitions by resonance sign (EIT or EIA) has developed in the mode of weak saturation of the atomic transition. The "dark" transitions are transitions of the type $F_g = F \to F_e = F$ and $F_g = F \to F_e = F - 1$ (where F_g and F_e are the total angular momenta of the atom in its ground and excited states, respectively), at which EIT resonances can be observed. In turn, the "bright" transitions are transitions of the type $F_g = F \to F_e = F + 1$ at which EIA can be observed. In particular, in the paper [13], this classification was theoretically justified in the framework of the perturbation theory for a two-frequency configuration composed of two co-propagating laser waves with arbitrary elliptical polarizations.

Recently, we generalized the results obtained in [13] to the case of a high field, when perturbation theory is inapplicable, in the model of stationary atoms [14]. In this work, we continued this study in the model of atomic gas (taking into account the Maxwellian distribution of atoms over velocities) using a density matrix formalism. As a result, the previously established classification of dipole transitions by the sign of the subnatural resonance was confirmed, regardless of the intensities of light waves.

II. Theoretical model and calculations

We consider the interaction of an elliptically polarized bichromatic field

$$\mathbf{E}(t,z) = E_1\mathbf{e}_1\,\mathrm{e}^{-i(\omega_1 t - k_1 z)} + E_2\mathbf{e}_2\,\mathrm{e}^{-i(\omega_2 t - k_2 z)} + \text{c.c.} \qquad (1)$$

with an atomic medium in which the ground (g) and excited (e) states that are degenerated with respect to the projections of the total angular momentum form a closed optical dipole transition $F_g \to F_e$ (Fig. 1). Here, $E_{1,2}$ and $\omega_{1,2}$ are the scalar amplitudes and frequencies of light waves, respectively, and $k_{1,2}$ are wavenumbers. We suppose that the frequency difference $\omega_1 - \omega_2$ is so insignificant that for the wavenumbers it can be assumed with a high degree of accuracy that $k_1 = k_2 = k$. The unit complex vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ of elliptic polarization we represented in a cyclic basis:

$$\mathbf{e}_j = \sum_{q=0,\pm1} e_j^{(q)}\mathbf{e}_q \quad (j = 1,2), \qquad (2)$$

where $\mathbf{e}_{\pm1} = \mp(\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$; $\mathbf{e}_0 = \mathbf{e}_z$ are the unit vectors of the cyclic basis; and $e_j^{(q)}$ are the contravariant components of the unit polarization vector of the jth wave. Let us direct the x axis along the principal axis of the polarization ellipse of the wave $\mathbf{E}_1$; then for unit polarization vectors (2) we have

$$\begin{aligned}\mathbf{e}_1 &= -\sin(\varepsilon_1 - \pi/4)\mathbf{e}_{-1} - \cos(\varepsilon_1 - \pi/4)\mathbf{e}_{+1},\\ \mathbf{e}_2 &= -\sin(\varepsilon_2 - \pi/4)e^{i\phi}\mathbf{e}_{-1} - \cos(\varepsilon_2 - \pi/4)e^{-i\phi}\mathbf{e}_{+1}.\end{aligned} \qquad (3)$$

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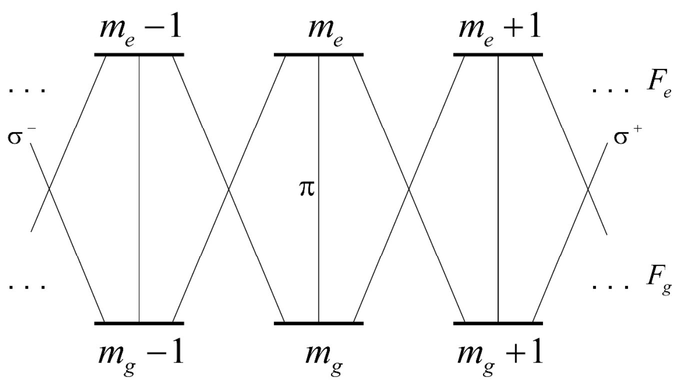

Fig. 1. Diagram of atomic energy levels degenerated with respect to the projections of the total angular momenta of the ground (F_g) and excited (F_e) states on the quantization axis z. Lines denote the light-induced transitions of σ^+-, σ^-- and π-types.

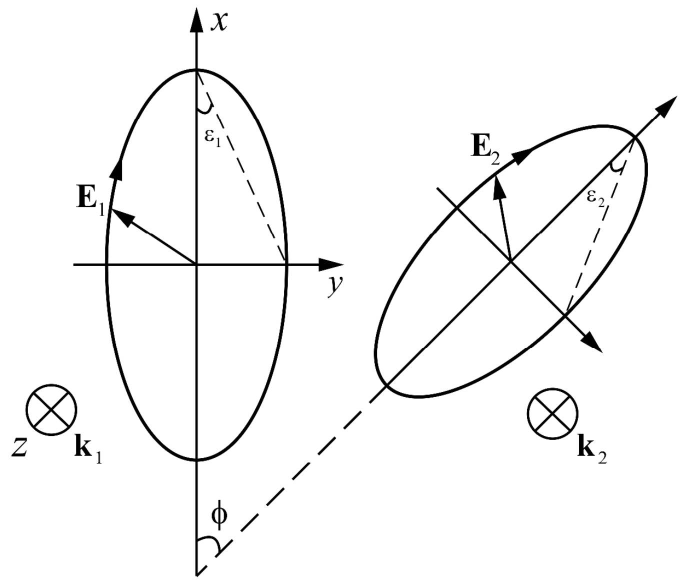

Fig. 2. Mutual orientation of polarization ellipses of the waves; $\boldsymbol{k}_1 = \boldsymbol{k}_2$ are the wave vectors of the waves, ϕ is the angle between the principal axes of the ellipses, and $\varepsilon_{1,2}$ are the ellipticity parameters.

Here, ϕ is the angle between the main axes of the polarization ellipses (Fig. 2); and the ellipticity parameter ε is defined in the interval $-\pi/4 \le \varepsilon \le \pi/4$, with $|\tan(\varepsilon)|$ being the ratio of the semiaxes of the ellipse, and the sign ε defining the direction of rotation of the electric component of the light field. In particular, $\varepsilon = \pm\pi/4$ и $\varepsilon = 0$ correspond to circular (right- and left-hand) and linear polarizations, respectively.

The atomic medium is supposed to be sufficiently rarefied, which allows us to neglect the effects of interatomic interaction and solve the problem in the single-atom approximation. For a mathematical description of the interaction of atoms with an electromagnetic field, we use the standard formalism of the density matrix $\hat{\rho}$. We separate the density matrix into four blocks,

$$\hat{\rho} = \hat{\rho}^{gg} + \hat{\rho}^{ee} + \hat{\rho}^{eg} + \hat{\rho}^{ge}, \tag{4}$$

where each block is a matrix

$$\hat{\rho}^{ab} = \sum_{m_a, m_b} \rho^{ab}_{m_a, m_b} \left| F_a, m_a \right\rangle \left\langle F_b, m_b \right|, \tag{5}$$

in the basis of Zeeman states $\left| F, m \right\rangle$; m are the projections of the angular momentum F onto the quantization axis z, running through the values $m = -F, -F+1, \ldots, F$. Since the density matrix is Hermitian, $\hat{\rho}^{gg\dagger} = \hat{\rho}^{gg}$, $\hat{\rho}^{ee\dagger} = \hat{\rho}^{ee}$, $\hat{\rho}^{eg\dagger} = \hat{\rho}^{ge}$. The diagonal matrix blocks $\hat{\rho}^{gg}$ and $\hat{\rho}^{ee}$ describe the populations of atomic states and low-frequency (Zeeman) coherences, and the off-diagonal matrix blocks $\hat{\rho}^{eg}$ and $\hat{\rho}^{ge}$ correspond to optical coherences.

Eliminating fast time oscillations at the frequency of one of the waves in the optical coherences (for example, at ω_1)

$$\hat{\rho}^{eg} = \hat{\tilde{\rho}}^{eg} e^{-i\omega_1 t}, \ \hat{\rho}^{ge} = \hat{\tilde{\rho}}^{ge} e^{i\omega_1 t}, \tag{6}$$

and using the resonance approximation, we reduce the optical Bloch equation for the density matrix to the following system of equations in the model of relaxation constants:

$$\begin{aligned}
&\left(\partial_t + \gamma_{opt} + \Gamma_0 - i\delta + ik\mathrm{v}_z\right)\hat{\tilde{\rho}}^{eg} = -iR_1\left[\hat{V}_1\hat{\rho}^{gg} - \hat{\rho}^{ee}\hat{V}_1\right] \\
&\quad - iR_2 e^{-i\Delta t}\left[\hat{V}_2\hat{\rho}^{gg} - \hat{\rho}^{ee}\hat{V}_2\right], \\
&\left(\partial_t + \gamma_{opt} + \Gamma_0 + i\delta - ik\mathrm{v}_z\right)\hat{\tilde{\rho}}^{ge} = -iR_1\left[\hat{V}_1^{\dagger}\hat{\rho}^{ee} - \hat{\rho}^{gg}\hat{V}_1^{\dagger}\right] \\
&\quad - iR_2 e^{i\Delta t}\left[\hat{V}_2^{\dagger}\hat{\rho}^{ee} - \hat{\rho}^{gg}\hat{V}_2^{\dagger}\right], \\
&\left(\partial_t + \gamma_{sp} + \Gamma_0\right)\hat{\rho}^{ee} = -iR_1\left[\hat{V}_1\hat{\tilde{\rho}}^{ge} - \hat{\tilde{\rho}}^{eg}\hat{V}_1^{\dagger}\right] \\
&\quad - iR_2\left[e^{-i\Delta t}\hat{V}_2\hat{\tilde{\rho}}^{ge} - e^{i\Delta t}\hat{\tilde{\rho}}^{eg}\hat{V}_2^{\dagger}\right], \\
&\left(\partial_t + \Gamma_0\right)\hat{\rho}^{gg} - \Gamma_0\hat{\rho}^{gg}(0) = \hat{\gamma}\{\hat{\rho}^{ee}\} - iR_1\left[\hat{V}_1^{\dagger}\hat{\tilde{\rho}}^{eg} - \hat{\tilde{\rho}}^{ge}\hat{V}_1\right] \\
&\quad - iR_2\left[e^{i\Delta t}\hat{V}_2^{\dagger}\hat{\tilde{\rho}}^{eg} - e^{-i\Delta t}\hat{\tilde{\rho}}^{ge}\hat{V}_2\right].
\end{aligned} \tag{7}$$

Here, $\Delta = \omega_2 - \omega_1$; $R_{1,2} = -dE_{1,2}/\hbar$ are the Rabi frequencies (d is the reduced matrix element of the dipole moment); γ_{sp} is the rate of radiation decay of the excited state; Γ_0 is the constant responsible for the time of flight or diffusion relaxation in the ground state to the initial (isotropic) distribution $\hat{\rho}^{gg}(0) = \hat{1}^{gg} \cdot \mathrm{Tr}\{\hat{\rho}\}/(2F_g+1)$ in the absence of a light field; $\hat{1}^{gg}$ is the identity matrix of dimension $(2F_g+1)\times(2F_g+1)$; $\mathrm{Tr}\{\ldots\}$ is the operation of computing the trace of the matrix; γ_{opt} is the relaxation rate of optical coherences; $\delta = \omega_1 - \omega_0$ is the detuning of the frequency ω_1 of one of the waves of the light field from the transition frequency

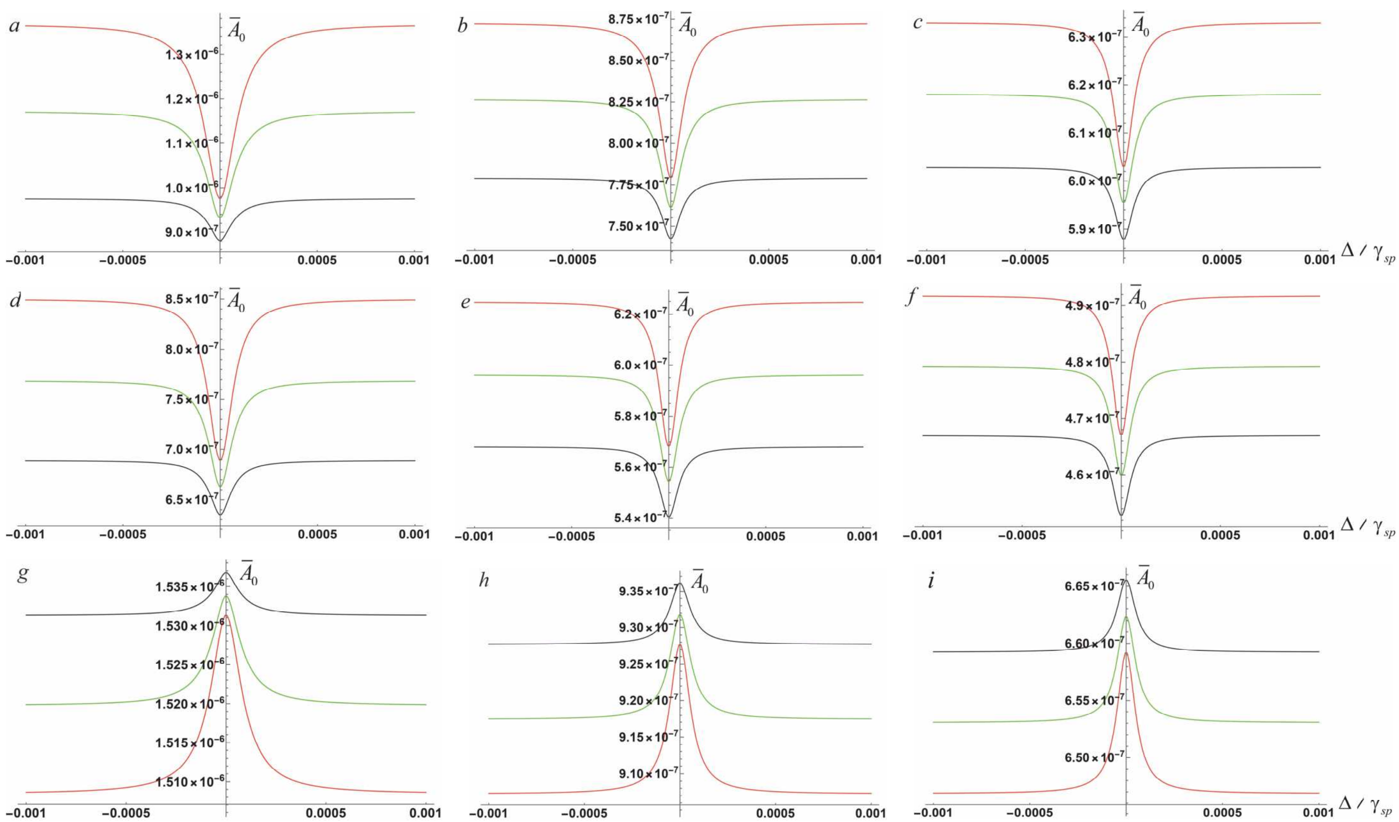

Fig. 3. Dependences of the signal $\bar{A}_0$ (15) on the wave frequency difference $\Delta = \omega_2 - \omega_1$ (in units of γ_{sp}) in a model with spontaneous transfer of anisotropy from the excited state to the ground state for the transitions (*a*) $F_g = 1 \rightarrow F_e = 1$, (*b*) $F_g = 2 \rightarrow F_e = 2$, (*c*) $F_g = 3 \rightarrow F_e = 3$, (*d*) $F_g = 2 \rightarrow F_e = 1$, (*e*) $F_g = 3 \rightarrow F_e = 2$, (*f*) $F_g = 4 \rightarrow F_e = 3$, (*g*) $F_g = 1 \rightarrow F_e = 2$, (*h*) $F_g = 2 \rightarrow F_e = 3$, and (*i*) $F_g = 3 \rightarrow F_e = 4$ at $\varepsilon_1 = \varepsilon_2 = 0$ (black curves), $\varepsilon_1 = \pi/8$, $\varepsilon_2 = -\pi/8$ (green curves), and $\varepsilon_1 = \pi/4$, $\varepsilon_2 = -\pi/4$ (red curves). Other model parameters include $R_1 = R_2 = 0.01\gamma_{sp}$, $\omega_D = 52\gamma_{sp}$, $\gamma_{opt} = 0.5\gamma_{sp}$, $\Gamma_0 = 0.5\cdot 10^{-4}\gamma_{sp}$, $\delta = 0$, $\phi = 0$.

ω_0; v_z is projection of the velocity of an atom onto an axis *z*;

$$\hat{V}_{1,2} = \hat{\mathbf{T}} \cdot \mathbf{e}_{1,2} = \sum_{q=0,\pm 1} \hat{T}_q e_{1,2}^{(q)} \tag{8}$$

are the dimensionless interaction operators. The cyclic components of the vector operator $\hat{\mathbf{T}}$ (the first-rank Wigner operators $\hat{T}_q$) are expressed in terms of 3*jm* symbols:

$$\hat{T}_q = \sum_{\{m\}} (-1)^{F_e - m_e} \begin{pmatrix} F_e & 1 & F_g \\ -m_e & q & m_g \end{pmatrix} \left| F_e, m_e \right\rangle \left\langle F_g, m_g \right|. \tag{9}$$

And $\hat{\gamma}\{\hat{\rho}^{ee}\}$ is an operator describing the arrival of atoms from an excited level to the ground level. In the standard spontaneous relaxation model with anisotropy transfer taken into account, we have

$$\hat{\gamma}\{\hat{\rho}^{ee}\} = \gamma_{sp}(2F_e + 1) \sum_{q=0,\pm 1} \hat{T}_q^{\dagger} \hat{\rho}^{ee} \hat{T}_q. \tag{10}$$

In the model without anisotropy transfer, another expression holds:

$$\hat{\gamma}\{\hat{\rho}^{ee}\} = \gamma_{sp} \frac{\hat{1}^{gg} \cdot \mathrm{Tr}\{\hat{\rho}^{ee}\}}{2F_g + 1}. \tag{11}$$

Note that for the cyclic transition $F_g \rightarrow F_e$, the total population at the ground and excited levels is conserved:

$$\mathrm{Tr}\{\hat{\rho}^{gg}\} + \mathrm{Tr}\{\hat{\rho}^{ee}\} = 1. \tag{12}$$

As a spectroscopic signal, we consider the light field absorption, which in the approximation of an optically thin medium is determined by the total population of the excited state:

$$A(t, k\mathrm{v}_z) = \mathrm{Tr}\{\hat{\rho}^{ee}(t, k\mathrm{v}_z)\}. \tag{13}$$

One can see that the right-hand sides of equations (7) are periodic functions of time with a period $T = 2\pi/|\Delta|$. According to the theorem on the existence of a dynamic steady state for the density matrix [15], the time dependence of the signal (13) is also periodic with the same period T. In this work, we find

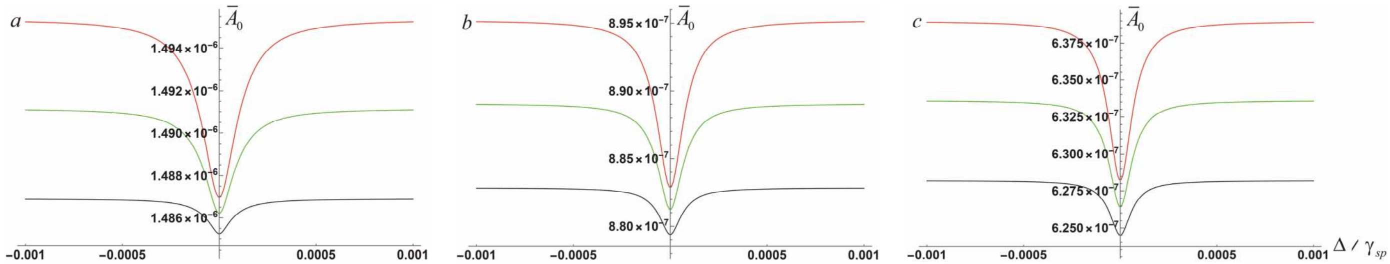

Fig. 4. Dependences of the signal $\bar{A}_0$ (15) on the wave frequency difference $\Delta = \omega_2 - \omega_1$ (in units of γ_{sp}) in a model without spontaneous transfer of anisotropy from the excited state to the ground state for the transitions (*a*) $F_g = 1 \to F_e = 2$, (*b*) $F_g = 2 \to F_e = 3$, (*c*) $F_g = 3 \to F_e = 4$ at $\varepsilon_1 = \varepsilon_2 = 0$ (black curves), $\varepsilon_1 = \pi/8$, $\varepsilon_2 = -\pi/8$ (green curves), and $\varepsilon_1 = \pi/4$, $\varepsilon_2 = -\pi/4$ (red curves). Other model parameters include $R_1 = R_2 = 0.01\gamma_{sp}$, $\omega_D = 52\gamma_{sp}$, $\gamma_{opt} = 0.5\gamma_{sp}$, $\Gamma_0 = 0.5 \cdot 10^{-4}\gamma_{sp}$, $\delta = 0$, $\phi = 0$.

zero harmonic of the signal (13), the expression for which has the following form:

$$A_0(k\mathrm{v}_z) = \frac{1}{T}\int_0^T A(t, k\mathrm{v}_z). \qquad (14)$$

For this purpose, we use the method of matrix continued fractions. Then, the expression (14) should be averaged over Maxwellian velocity distribution of atoms:

$$\bar{A}_0 = \frac{k}{\sqrt{\pi}\,\omega_D}\int_{-\infty}^{+\infty} A_0(k\mathrm{v}_z)\cdot e^{-\frac{(k\mathrm{v}_z)^2}{\omega_D^2}}\, d(\mathrm{v}_z), \qquad (15)$$

where ω_D is the Doppler width of a spectral line.

In this paper we investigate the signal (15) as a function of the frequency difference of the waves Δ. The subnatural resonance structure appears near $\Delta = 0$. Numerical calculations were carried out for various elliptical polarizations of the waves (including linear and circular) and at the condition $|R_1|^2 + |R_2|^2 > \gamma_{sp}\Gamma_0$, i.e., for a sufficiently strong field intensity, when the perturbation theory [13] is no longer valid. We considered a model with spontaneous transfer of anisotropy (10) (Fig. 3) and without it (11) (Fig. 4).

Figures 3(*a*, *b*, *c*) and 3(*d*, *e*, *f*) show the dependences of the signal (15) on Δ at the condition (10) for particular cases of transitions $F_g = F \to F_e = F$ and $F_g = F \to F_e = F - 1$, respectively. One can see that these resonances are directed downwards, and the EIT effect is observed. Therefore, these transitions are 'dark' ones. Figures 3(*g*, *h*, *i*) show similar dependences for particular cases of transitions $F_g = F \to F_e = F + 1$. In this case, the resonances are directed upwards, i.e., the EIA effect takes place; and therefore, these transitions are 'bright' ones. However, if spontaneous anisotropy transfer is absent (11), the resonances are already directed downwards (in contrast to the results in Figs. 3(*g*, *h*, *i*)) for transitions $F_g = 1 \to F_e = 2$ (Fig. 4*a*), $F_g = 2 \to F_e = 3$ (Fig. 4*b*), $F_g = 3 \to F_e = 4$ (Fig. 4*c*), i.e., EIT resonances are formed. Thus, in the model of atomic gas the ellipticity of the waves does not affect the sign of the subnatural resonance in the two-frequency configuration of the light field at strong intensities of waves, and this sign is determined only by the angular momenta F_g and F_e. The formation of the EIA resonance is associated with the spontaneous transfer of anisotropy from the excited state to the ground one.

III. Conclusions

We have theoretically researched the type of the subnatural resonance at the closed dipole transition outside of the framework of the perturbation theory. As a model, we have considered the interaction between a two-level atomic gas degenerated with respect to the projections of the total angular momentum and the two-frequency field of two co-propagating waves with arbitrary elliptical polarizations. The sign of resonance (EIT or EIA) has been shown not to depend on the parameters of ellipticity and intensity of the field. The EIA resonance arises due to the spontaneous transfer of anisotropy from the excited state of the atom to the ground state at the transition $F_g = F \to F_e = F + 1$. As a result, we have generalized and verified the previous classification of cyclic dipole transitions in the direction of the subnatural resonance for high intensities of light waves. 'Dark' transitions are the transitions $F_g = F \to F_e = F$ and $F_g = F \to F_e = F - 1$, at which it is possible to observe the EIT resonance. In turn, the 'bright' transitions include the transitions $F_g = F \to F_e = F + 1$, for which the EIA resonance is formed.

Acknowledgments

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